PROBLEM 14: EVOLUTION OF FLATNESS *(15 points)*

The following problem was Problem 3, Quiz 3, 2004.

The "flatness problem" is related to the fact that during the evolution of the standard cosmological model, Ω is always driven away from 1.

(a) *(9 points)* During a period in which the universe is matter-dominated (meaning that the only relevant component is nonrelativistic matter), the quantity

$$\frac{\Omega - 1}{\Omega}$$

grows as a power of t, provided that Ω is near 1. Show that this is true, and derive the power. (Stating the right power without a derivation will be worth 3 points.)

(b) *(6 points)* During a period in which the universe is radiation-dominated, the same quantity will grow like a different power of t. Show that this is true, and derive the power. (Stating the right power without a derivation will again be worth 3 points.)

In each part, you may assume that the universe was *always* dominated by the specified form of matter.

(a) $H^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$ (curvature)

ρ_c : $H^2 = \frac{8\pi G}{3}\rho_c$

$\Omega \equiv \rho/\rho_c$

$\Rightarrow \frac{8\pi G}{3}\rho_c = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$

$\Rightarrow \frac{8\pi G}{3}(\rho_c - \rho) = -\frac{kc^2}{a^2}$

$\Rightarrow \rho - \rho_c = \frac{3kc^2}{8\pi G a^2}$ / $\frac{1}{\rho}$

$\Rightarrow \frac{\rho - \rho_c}{\rho} = \frac{3kc^2}{8\pi G a^2 \rho}$

$\frac{\rho/\rho_c - 1}{\rho/\rho_c} = \frac{\Omega - 1}{\Omega} \Rightarrow \frac{\Omega - 1}{\Omega} = \frac{3kc^2}{8\pi G a^2 \rho}$

If we have dust-like matter, $\rho \propto 1/a^3$

$\Rightarrow \left.\frac{\Omega - 1}{\Omega}\right|_{matter} \propto \frac{1}{a^2} a^3 = a$

If matter dominates the cosmic evolution, then $a \propto t^{2/3}$

(b) For radiation, $\rho \propto 1/a^4$ $\Rightarrow$ $\left.\frac{\Omega-1}{\Omega}\right|_{rad} \propto \frac{1}{a^2} a^4 = a^2$

In radiation domination, $a \propto t^{1/2}$

$\Rightarrow$ $\left.\frac{\Omega-1}{\Omega}\right|_{rad} \propto t$ //